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COMMENT

Comment on 'Note on generalization of Shannon theorem and inequality'

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Abstract
Kwork Sau Fa (1998 *J. Phys. A: Math. Gen.* **31** 8159) has shown an inequality '$S(AB) \leqslant S(A) + S(B) + (1-q)S(A)S(B)$' for two interacting systems A and B. A typical example of $S(A)$ is the Tsallis entropy as stated in his paper. However, there exist many counterexamples to the above inequality. The reason leading to the incorrect result is also presented.

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In [1], the following inequality is proved for two interacting systems A and B,

$$S(AB) \leqslant S(A) + S(B) + (1-q)S(A)S(B) \tag{1}$$

where $S(A)$ is the generalized Tsallis entropy derived in [1]. A typical example of $S(A)$ is the Tsallis entropy defined by

$$S_q^T = k\frac{1-\sum_i p_i^q}{q-1} \tag{2}$$

as stated in (2.19) and the conclusion in [1]. In this comment, we consider the inequality (1) in case $S = S_q^T$.

This short report shows that the inequality (1) does *not* hold in general. In fact, there exist many counterexamples to the inequality (1). The simplest counterexample is shown as follows.

Let A, B be two random variables taking two values 0 and 1. Then, consider the following simple joint distribution:

$$p(A=0, B=0) = p(1-x) \qquad p(A=0, B=1) = (1-p)y \tag{3}$$

$$p(A=1, B=0) = px \qquad p(A=1, B=1) = (1-p)(1-y) \tag{4}$$

where

$$0 \leqslant p, x, y \leqslant 1. \tag{5}$$

Obviously the probability $p(A, B)$ ((3) and (4)) satisfies the conditions of joint distribution.

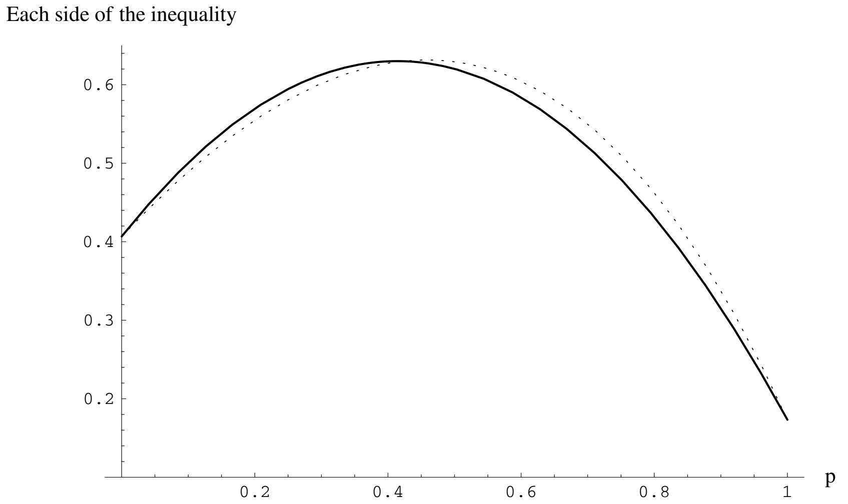

Figure 1. Graph showing each side of the inequality (1). (—: left-hand side of (1), – – –: right-hand side of (1)).

For the above joint distribution $p(A, B)$ given by (3) and (4), we can compute the marginal distribution as follows:

$$p(A=0) = p(1-x) + (1-p)y \qquad p(A=1) = px + (1-p)(1-y) \tag{6}$$

$$p(B=0) = p(1-x) + px = p \qquad p(B=1) = 1-p. \tag{7}$$

Clearly, there exist many $p, x, y \in [0, 1]$ such that

$$p(A, B) \neq p(A)p(B) \tag{8}$$

$$\Leftrightarrow \quad p(A=i, B=j) \neq p(A=i)p(B=j) \qquad (i, j \in \{0, 1\}). \tag{9}$$

This means that the two systems A and B interact with each other. Paper [1] states that under the condition (8) the inequality (1) holds.

Let the left- and right-hand sides of (1) be $L_q^T(A, B)$ and $R_q^T(A, B)$, respectively.

$$L_q^T(A, B) \equiv S_q^T(AB) \tag{10}$$

$$R_q^T(A, B) \equiv S_q^T(A) + S_q^T(B) + (1-q)S_q^T(A)S_q^T(B) \tag{11}$$

where we consider the case $S = S_q^T$ in the inequality (1) as stated before.

When $x = 0.1$, $y = 0.7$ and $q = 2.1$, $L_q^T(A, B)$ and $R_q^T(A, B)$ can be plotted as a function of the probability p in one graph (figure 1). The solid line and dashed line in the graph shown in figure 1 represent $L_q^T(A, B)$ and $R_q^T(A, B)$, respectively. This example shows that the inequality (1) proved in [1] does not hold in general.

$$S_q^T(AB) \not\leq S_q^T(A) + S_q^T(B) + (1-q)S_q^T(A)S_q^T(B). \tag{12}$$

There exist such many counterexamples to the inequality (1). The reason leading to this incorrect result is his following statement after (3.3) in [1]: 'g_j is as the total probability $\sum_i p_i^q g_{ij}$ of finding the event B_j in system B'. g_j is not a probability because $\sum_j g_j = \sum_j \sum_i p_i^q g_{ij} \neq 1$ for any $q \in R^+$.

References

[1] Fa K S 1998 *J. Phys. A: Math. Gen.* **31** 8159